Hypersonic Interactions with Surface Mass Transfer—Part I: Steady Flow

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Abstract

THE steady hypersonic interaction problem with air, argon, and helium injection is analyzed. A complete solution has been obtained for the basically nonsimilar problem of a slender wedge wing at an angle of attack. The results indicate that a lighter, gas-like helium may be less appealing as a coolant due to the large induced pressures.

Contents

Most of the earlier works connected with the study of surface mass transfer are pertinent to flights at supersonic speeds, where the corresponding viscous-inviscid interaction problem is not important, or have treated this problem under the restrictive category of similar solutions as reviewed in Ref. 1. Gupta et al.[2] recently presented a first-order correction to the strong and weak interaction-induced pressures with vectored injection of a different fluid for a nonsimilar flow problem. In the present work, an exact solution has been obtained for the complete hypersonic interaction problem[3] with surface mass transfer (involving normal injection or suction) for the case of a slender wedge wing at an angle of attack. The analysis is carried out for a perfect gas with constant specific heat C_p, specific heat ratio γ, Lewis number Le, and the Prandtl number Pr (an asterisk denotes dimensional quantities). The transformed boundary-layer momentum, energy, and species conservation equations are written, in terms of the dimensionless stream function f, dimensionless total enthalpy H (equal to $2H^*/u_\infty^{*2}$, where u_∞^* is a freestream velocity component in the $x^*=xL^*$ streamwise direction), and mass fraction of the freestream species c_f [equal to $(1-c_i)$, where c_i is the mass fraction of the injected species ρ_i^*/ρ^*] as follows[1]:

$$4x\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial x\partial\eta}-\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial x}\right)-f\frac{\partial^2 f}{\partial\eta^2}-\bar{p}\frac{\partial}{\partial\eta}\left(\lambda\frac{\partial^2 f}{\partial\eta^2}\right)$$
$$+\beta\left(\frac{2x}{\bar{p}}\frac{\partial\bar{p}}{\partial x}-1\right)\left[H-\left(\frac{\partial f}{\partial\eta}\right)^2\right]\left[\frac{s-c_f(s-1)}{r-c_f(r-1)}\right]=0 \quad (1)$$

$$4x\left(\frac{\partial f}{\partial\eta}\frac{\partial H}{\partial x}-\frac{\partial H}{\partial\eta}\frac{\partial f}{\partial x}\right)-f\frac{\partial H}{\partial\eta}-\bar{p}\frac{\partial}{\partial\eta}\left[\frac{\lambda}{Pr}\left\{\frac{\partial H}{\partial\eta}\right.\right.$$
$$+2(Pr-1)\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta^2}-(Le-1)$$
$$\left.\left.\times\left[\frac{(r-1)}{(1-c_f)(r-1)+1}\right]\left[H-\left(\frac{\partial f}{\partial\eta}\right)^2\right]\frac{\partial c_f}{\partial\eta}\right\}\right]=0 \quad (2)$$

$$4x\left(\frac{\partial f}{\partial\eta}\frac{\partial c_f}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial c_f}{\partial\eta}\right)-f\frac{\partial c_f}{\partial\eta}=\bar{p}\frac{\partial}{\partial\eta}\left(\frac{\lambda}{Pr}Le\frac{\partial c_f}{\partial\eta}\right) \quad (3)$$

$$\eta=\frac{1}{2}\left(\frac{Re_{\infty,L^*}}{Cp_0\bar{\chi}_{L^*}}\right)^{1/2}x^{-1/4}\int_{y_w}^{y}\rho\,dy,$$
$$f=\left(\frac{Re_{\infty,L^*}}{Cp_0\bar{\chi}_{L^*}}\right)^{1/2}x^{-1/4}\psi,\qquad \lambda=\frac{\mu\rho}{Cp}$$

where $p=p^*/p_\infty^*$ is the pressure, $p_0\bar{\chi}$ the induced pressure obtained from the zero-order strong interaction theory,[3] $\bar{\chi}=M_\infty^3\sqrt{(C/Re_{\infty,x^*})}$ the interaction parameter, C the Chapman-Rubesin constant, Re the Reynolds number, $r=C_{pi}/C_{pf}$ the injected-to-freestream specific-heats ratio, $s=M_f/M_i$ the ratio of molecular weights, $\mu=\mu^*/\mu_\infty^*$ the viscosity, $\rho=\rho^*/\rho_\infty^*$ the density, $y=y^*/L^*$ the normal coordinate, ψ the stream function, and subscript w denotes evaluation at the wedge surface. The normalized pressure function $\bar{p}(x)$ appearing in Eqs. (1-3) has been defined as

$$\bar{p}(x)=p(x)/p_0\bar{\chi} \quad (4)$$

By retaining pressure as a coefficient of higher-order terms in Eqs. (1-3), the character of the interaction problem has been distinctly brought out.

The pressure function $\bar{p}$ has been evaluated by using the relations[1]

$$\bar{p}=\frac{\bar{I}}{\bar{\delta}};\quad \bar{I}(x)=\int_0^e\left[\frac{c_f+(1-c_f)s}{c_f+(1-c_f)r}\right]\left[H-\left(\frac{\partial f}{\partial\eta}\right)^2\right]d\eta\Big/I_0 \quad (5)$$

and the normalized[1] boundary-layer thickness $\bar{\delta}$ has been obtained from the equation

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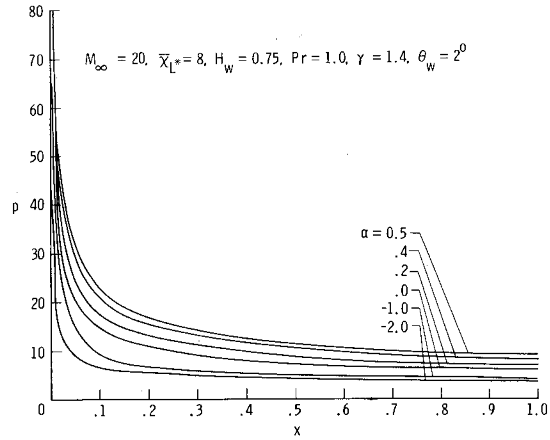

Fig. 1 Variation of induced pressure along the plate for different values of the injection parameter.

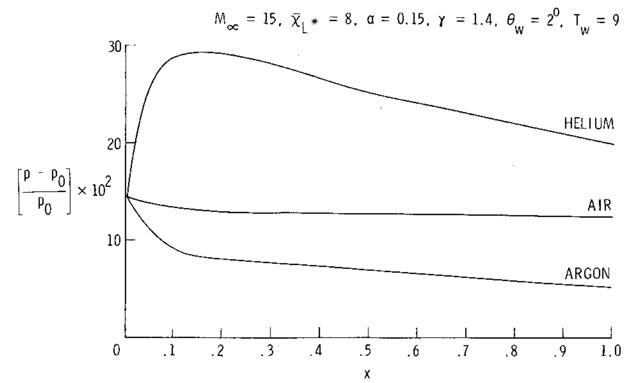

Fig. 2 Percent changes in induced pressure with surface injection of various gases.

$$x\frac{d\bar{\delta}}{dx} - \frac{3}{4}\bar{\delta} - \frac{\bar{s}(x)}{\xi} + \frac{K_w x^{1/4}}{\xi} = 0 \qquad (6)$$

with

$$\xi = \delta_0 \bar{x}_{L^*}^{-1/2}, \quad \bar{s}(x) = x^{1/4} K(x), \quad K = K_w + M_\infty \frac{d\delta}{dx}$$

and δ_0 and I_0 are constants associated with zero-order series solution.[1] Equation (6) has been obtained by inverting the tangent wedge expression.[3]

The appropriate initial and boundary conditions with surface mass transfer for the solution of Eqs. (1-3) are provided in detail in Ref. 1.

The method of obtaining the pressure distribution $\bar{p}(x)$ from the solution of boundary-layer equations (1-3) can now be described as follows: From the solution of the boundary-layer equations giving $f(x,\eta)$, $H(x,\eta)$, and $c_f(x,\eta)$, we calculate the $\bar{I}(x)$ distribution using Eq. (5); Eq. (6) is now solved for $\bar{\delta}(x)$ and $\bar{p}(x)$ is obtained from Eq. (5). It may be mentioned here that Eq. (6) is solved by employing a value of $\bar{\delta}(x)$ from the previous iteration in the expression for $\bar{s}(x)$.

The numerical solution to the boundary-layer equations (1-3) has been obtained with a finite difference method,[4] by converting these equations into a set of linear algebraic equations through the application of implicit finite difference formulas.

In Fig. 1 is shown the effect of varying the injection/suction parameter, α, on the distribution of p along the plate surface for the flow conditions given in the figure. Here θ_w is the angle of inclination of either side of the wedge. With air injection (i.e., for positive values of α), p increases due to the larger displacement of the external flow as compared to the no-injection case ($\alpha = 0$), whereas with suction the opposite is true.

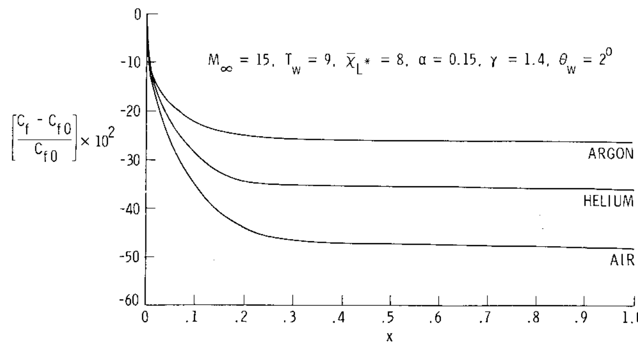

Fig. 3 Percent changes in local skin friction coefficient with surface injection of various gases.

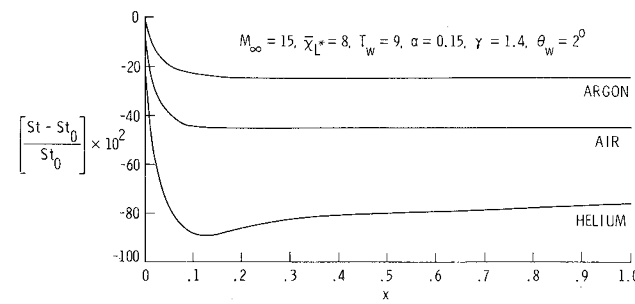

Fig. 4 Percent changes in Stanton number with surface injection of various gases.

Figures 2-4 show the percent changes in the values of the induced pressure p, local skin friction coefficient C_f, and the Stanton number St, respectively. The values with a zero subscript are no-injection values. Larger pressures are induced with injection of a lighter gas-like helium due to the increased displacement of the outer flow. There is a greater reduction in the local skin friction coefficient with injection of air, whereas the reduction in the Stanton number is maximum with helium injection. Since the expression for St involves $\bar{p}$ as well as the concentration gradient, no direct reasons may be assigned for the different behavior of these gases in controlling p, C_f, and St. With the injection of helium there is decreased mixture density at the surface with increasing helium concentration, and this tends to decrease the skin friction. However, due to the larger induced pressures with helium injection, the effect of decreased mixture density is more than compensated when compared to the air injection. In the case of Stanton number, however, the specific heat of helium is much greater than air, and the concentration gradient contributes to a heat-flux away from the wall with helium injection. Therefore, even with larger induced pressures in this case, there is more reduction in heating as compared to the case with air injection.

More detailed results including the lift and drag coefficients, the lift-to-drag ratio, the position of the aerodynamic center, etc., are provided in Ref. 1.

References

[1] Gupta, R. N., Varghese, N. K., Rodkiewicz, C. M., and Jain, A. C., "Hypersonic Interactions with Surface Mass Transfer—Part I: Steady Flow over a Slender Wedge Wing," AIAA Paper 82-0979, St. Louis, Mo., June 1982.

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